

Electron collisions with Beryllium and its ions

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Introduction

Motivation

- The primary motivation is to provide accurate atomic collision data for science and industry:
 - Astrophysics
 - Fusion research
 - Lighting industry
 - Medical and materials applications
- Provide a rigorous foundation for collision theory with long-ranged (Coulomb) potentials.

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Distant history

- Prior to the 1990s theory and experiment generally did not agree for:
 - electron-hydrogen excitation or ionisation,
 - electron-helium excitation or ionisation,
 - single or double photoionisation of helium.
- Have developed the convergent close-coupling (CCC) theory for electron/positron/photon/(anti)proton collisions with atoms/ions/molecules that is applicable at all energies for the major excitation and ionisation processes.

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 - Two-body problems
 - Three-body problems
- Have developed a surface integral approach to scattering theory that is valid for short- and long-range potentials
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Non-relativistic CCC theory

target structure and scattering

- Structure: using the Laguerre basis $\xi_{nl}^{(\lambda)}(r)$ write:
 - “one-electron” (Be^{3+} and Be^+) states:

$$\phi_{nl}^{(\lambda)}(r) = \sum_{n'} C_{nl}^{n'} \xi_{n'l}^{(\lambda)}(r)$$
 - “two-electron” (Be^{2+} and Be) states:

$$\phi_{nls}^{(\lambda)}(r_1, r_2) = \sum_{n', n''} C_{nls}^{n' n''} \xi_{n'l'}^{(\lambda)}(r_1) \xi_{n''l''}^{(\lambda)}(r_2).$$
 - Coefficients C are obtained by diagonalising the target (FCHF) Hamiltonian

$$\langle \phi_f^{(\lambda)} | H_T | \phi_i^{(\lambda)} \rangle = \varepsilon_f^{(\lambda)} \delta_{fi}. \quad (1)$$

Scattering:

- Electron-atom wavefunction is expanded as

$$|\Psi_i^{(+)}\rangle \approx \mathcal{A} \sum_{n=1}^N |\phi_n^{(\lambda)}\rangle \langle \phi_n^{(\lambda)} | \psi_i^{(+)}\rangle \equiv \mathcal{A} I_N |\psi_i^{(+)}\rangle. \quad (2)$$

- Solve for $T_{fi} \equiv \langle \mathbf{k}_f \phi_f^{(\lambda)} | V | \Psi_i^{(+)}\rangle$ at $E = \varepsilon_i^{(\lambda)} + \epsilon_{k_f}$,

$$\begin{aligned} \langle \mathbf{k}_f \phi_f^{(\lambda)} | T | \phi_i^{(\lambda)} \mathbf{k}_i \rangle &= \langle \mathbf{k}_f \phi_f^{(\lambda)} | V | \phi_i^{(\lambda)} \mathbf{k}_i \rangle \\ &+ \sum_{n=1}^N \int d^3k \frac{\langle \mathbf{k}_f \phi_f^{(\lambda)} | V | \phi_n^{(\lambda)} \mathbf{k} \rangle \langle \mathbf{k} \phi_n^{(\lambda)} | T | \phi_i^{(\lambda)} \mathbf{k}_i \rangle}{E + i0 - \varepsilon_n^{(\lambda)} - \epsilon_k}. \end{aligned} \quad (3)$$

- Have step-function behaviour:

$$\langle \mathbf{k}_f \phi_f^{(\lambda)} | T | \phi_i^{(\lambda)} \mathbf{k}_i \rangle \approx 0 \text{ for } \varepsilon_f^{(\lambda)} > \epsilon_{k_f}.$$

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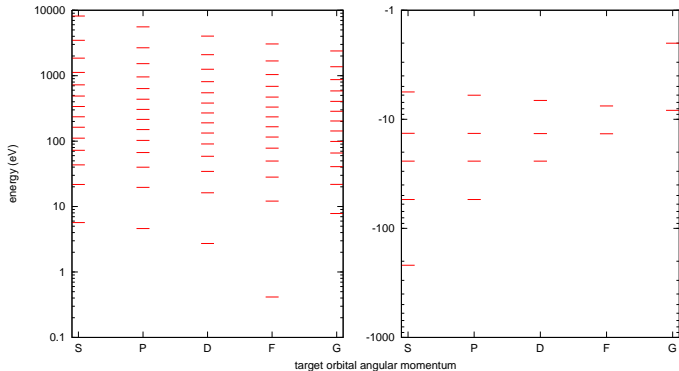
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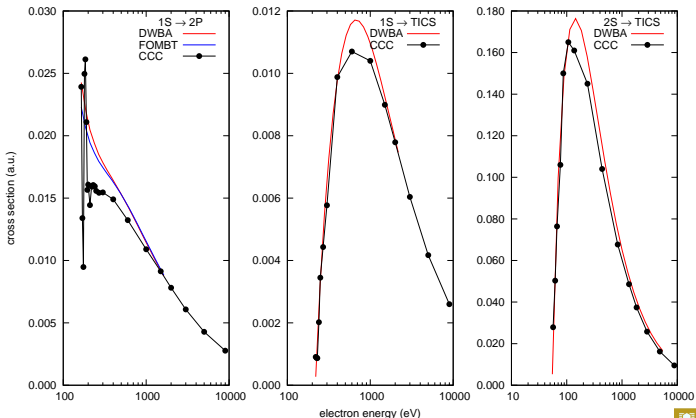
e^- -Be³⁺ collisions

- Energy levels in the CCC(85) calculations



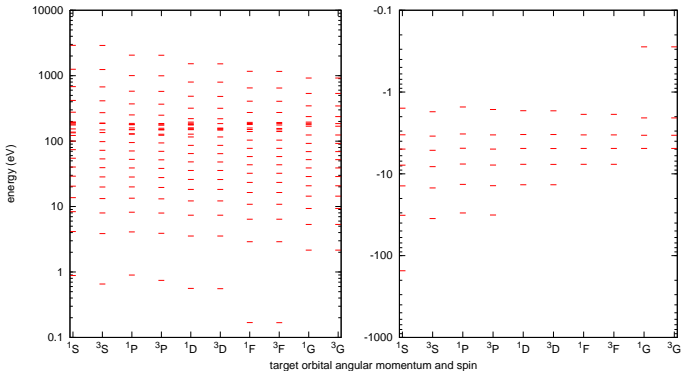
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● Cross Sections



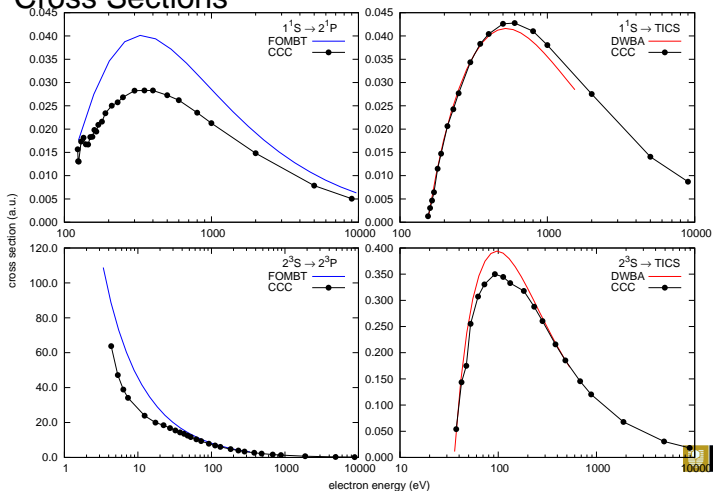
e^- -Be²⁺ collisions

- Energy levels in the CCC(284) calculations



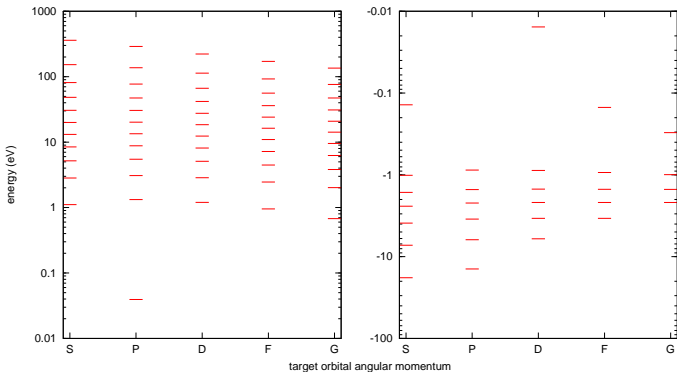
e^- -Be²⁺ collisions

Cross Sections



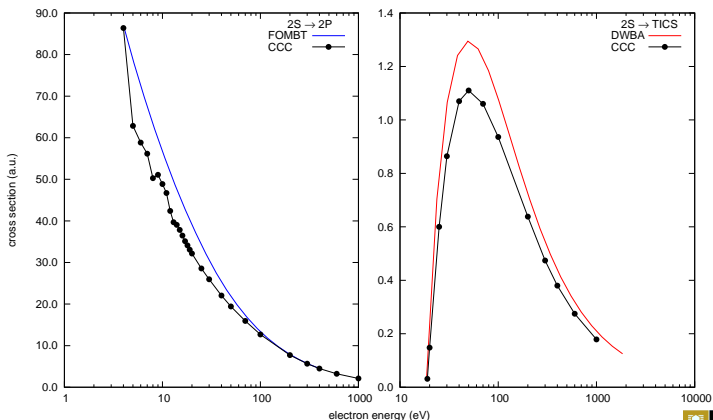
e^- -Be⁺ collisions

- Energy levels in the CCC(84) calculations



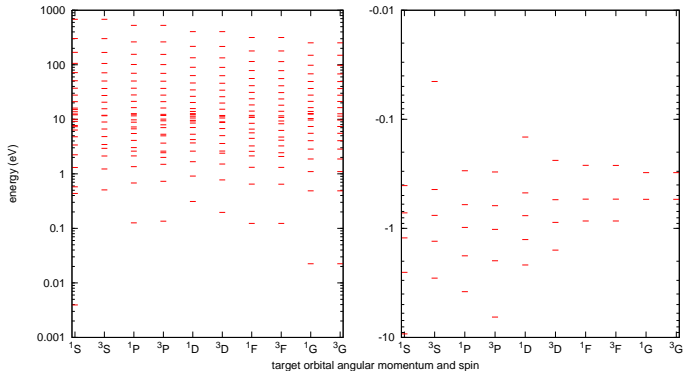
e^- -Be⁺ collisions

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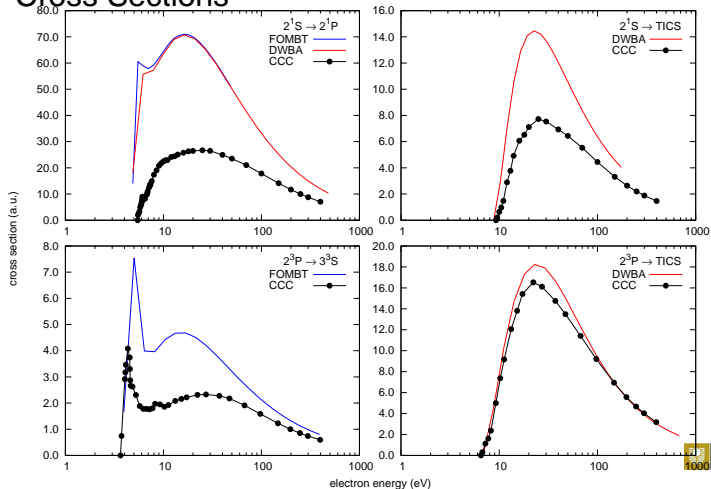
e⁻-Be collisions

● Energy levels in the CCC(292) calculations



e^- -Be collisions

● Cross Sections



Concluding Remarks

- CCC method is accurate for e^- -Be $^{q+}$ collisions, where $q = 0, 1, 2, 3$
- Comparison with 1st-order theories shows occasional substantial discrepancies
- For each q need to determine the projectile energy range, and the number of initial states of interest
- Resonance regions need a dense energy mesh
- Comparison with non-perturbative approaches would be welcome